CS 4440 A

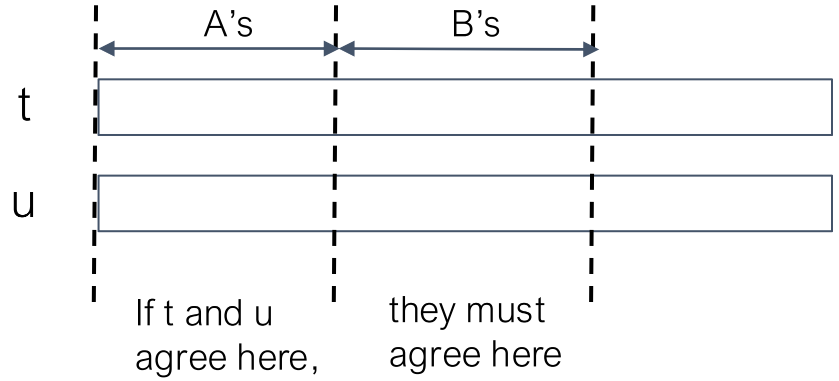
Emerging Database Technologies

Lecture 4
01/26/26

Recap: Functional dependency (FD)

Definition: if two tuples of R agree on all the attributes $A_1, A_2, \ldots, A_n$, they must also agree on (or functionally determine) $B_1, B_2, \ldots, B_m$

- Denoted as $A_1A_2 \ldots A_n \rightarrow B_1B_2 \ldots B_m$



A->B means that "whenever two tuples agree on A then they agree on B."

Recap: Closure of attributes

Given a set of attributes $A_1, \ldots, A_n$ and a set of FDs F, the closure, $\{A_1, \ldots, A_n\}^+$ is the set of attributes B where $\{A_1, \ldots, A_n\} \rightarrow B$ follows from the FDs in F

$AB \rightarrow C$

$BC \rightarrow AD$

$D \rightarrow E$

$CF \rightarrow B$

{A, B}+

A, B, C, D, E

Cannot be expanded further, so this is a closure

Recap: Keys and Superkeys

A superkey is a set of attributes $A_1, \ldots, A_n$ s.t.
for any other attribute B in R,
we have $\{A_1, \ldots, A_n\} \rightarrow B$

i.e. all attributes are functionally determined by a superkey

A key is a minimal superkey

This means that no subset of a key is also a superkey
(i.e., dropping any attribute from the key makes it no longer a superkey)

Back to Design Theory

Now that we know how to find FDs, it’s a straight-forward process:

1. Search for “bad” FDs

2. If there are any, then *keep decomposing the table into sub-tables* until no more bad FDs

3. When done, the database schema is *normalized*

Recall: there are several normal forms…

Normal Forms

1st Normal Form (1NF) = All tables are flat

2nd Normal Form = disused

Boyce-Codd Normal Form (BCNF)

3rd Normal Form (3NF)

$\Bigg\}$ DB designs based on functional dependencies, intended to prevent *data anomalies*

Our focus in this lecture

4th and 5th Normal Forms = see text books

Agenda

1. Boyce-Codd Normal Form
2. Properties of Decomposition
3. 3NF
4. MVDs

1. BCNF

Boyce-Codd Normal Form (BCNF)

Main idea is that we define “good” and “bad” FDs as follows:

- X → A is a *“good FD” if X is a (super)key*
  - In other words, if A is the set of all attributes
- X → A is a *“bad FD”* otherwise

We will try to eliminate the “bad” FDs!

Boyce-Codd Normal Form (BCNF)

Why does this definition of “good” and “bad” FDs make sense?

- X → A: each value of X is associated with exactly one value of A

If X is *not* a (super)key, it functionally determines *some* of the attributes; therefore, those other attributes can be duplicated

- Recall: this means there is <u>redundancy</u>
- And redundancy like this can lead to data anomalies!

“bad FD”: **Position → Phone**

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

Boyce-Codd Normal Form

BCNF is a simple condition for removing anomalies from relations:

A relation R is <u>in BCNF</u> if:

if $\{A_1, \ldots, A_n\} \rightarrow B$ is a non-trivial FD in R

then $\{A_1, \ldots, A_n\}$ is a superkey for R

Equivalently: $\forall$ sets of attributes X, either ($X^+ = X$) or (X^+ = all attributes)

In other words: there are no “bad” FDs

Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

SSN → Name,City

This FD is bad because it is not a superkey

⟹ Not in BCNF

What is the key?

{SSN, PhoneNumber}

Example

Name	SSN	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Madison

SSN	PhoneNumber
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121
987-65-4321	908-555-1234

SSN → Name,City

This FD is now good because it is the key

Let's check anomalies:
- Redundancy?
- Update?
- Delete?

Now in BCNF!

Boyce-Codd Normal Form (BCNF)

Special case: Any two-attribute relation is in BCNF

- If there are no nontrivial FDs, BCNF holds
- If A → B holds, but not B → A, the only nontrivial FD has A (i.e., the key) on the left
- Symmetric case when B → A holds, but not A → B
- If both A → B and B → A hold, any nontrivial FD has A or B (both are keys) on the left

Employee(empID, SSN)

empID → SSN
SSN → empID

BCNF Decomposition Algorithm

BCNFDecomp(R):

- Find an FD $X \rightarrow Y$ that violates BCNF
  (X and Y are sets of attributes)
- Compute the closure X+
- <u>let</u> $Y = X^+ - X$, $Z = (X^+)^C$

Let Y be the attributes that X functionally determines (+ that are not in X)

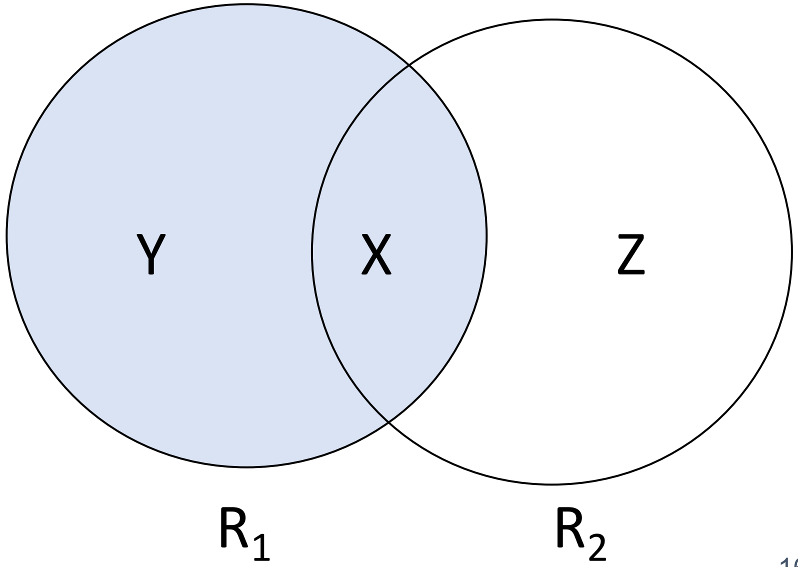
And let Z be the complement, the other attributes that it doesn't

BCNF Decomposition Algorithm

BCNFDecomp(R):

- Find an FD $X \rightarrow Y$ that violates BCNF
  (X and Y are sets of attributes)
- Compute the closure $X+$
- let $Y = X^+ - X$, $Z = (X^+)^C$
  decompose R into $R_1(X \cup Y)$ and $R_2(X \cup Z)$

Split into one relation (table) with X plus the attributes that X determines (Y)…

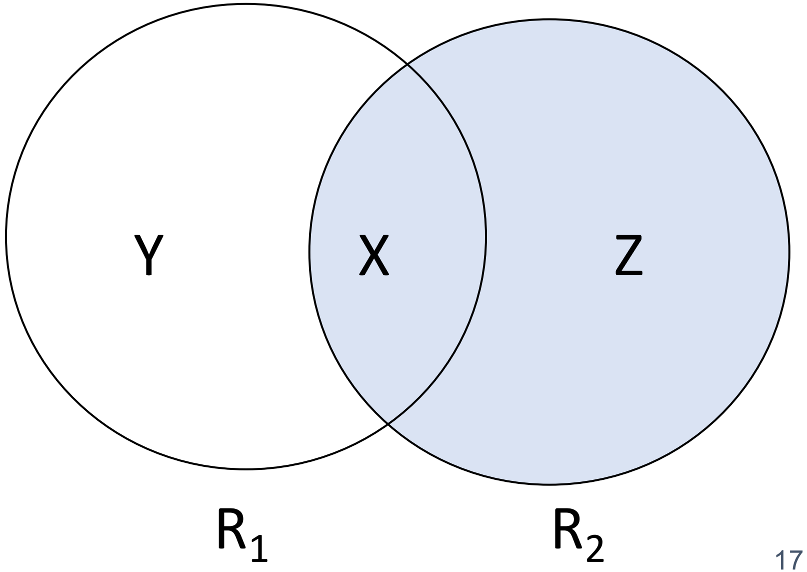


BCNF Decomposition Algorithm

BCNFDecomp(R):

- Find an FD $X \rightarrow Y$ that violates BCNF
  (X and Y are sets of attributes)
- Compute the closure X+
- let $Y = X^+ - X$, $Z = (X^+)^C$
  decompose R into $R_1(X \cup Y)$ and $R_2(X \cup Z)$
- Recursively decompose R_1 and R_2

And one relation with X plus the attributes it does not determine (Z)



Note: Projection of FDs

Let F be the set of FDs in the relation R. What FD's hold for $R_1 = \pi_L(R)$?

An FD $X \rightarrow Y$ from the original relation R will hold in the project R_1 iff

- Attributes in X and Y are all contained with R_1
- $X \rightarrow Y$ is logical implied by the original set F

Example

- Suppose R(A, B, C, D) has FDs $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow D\}$
- Then the FD's for R_1(A, C, D) are
  - $A \rightarrow C$: Implied by F
  - $C \rightarrow D$: Inherited from F

Example: BCNF Decomposition

What is R’s key?

- In general, there can be multiple decompositions

```
R(title,year,studioName,president,presAddr)
```

R’s FDs

title year → studioName
studioName → president
president → presAddr

Example: BCNF Decomposition

- In general, there can be multiple decompositions

```
R(title,year,studioName,president,presAddr)
```

Key

R's FDs

BCNF violations

title year → studioName
studioName → president
president → presAddr

Example: BCNF Decomposition

- In general, there can be multiple decompositions

```
R(title,year,studioName,president,presAddr)
```

```
R1(studioName,president,presAddr)
```

```
R2(title,year,studioName)
```

Key

R's FDs

title year → studioName

BCNF violations

studioName → president

president → presAddr

R2's FDs

title year → studioName

Is R2 in BCNF?

Example: BCNF Decomposition

- In general, there can be multiple decompositions

`R(title,year,studioName,president,presAddr)`

`R1(studioName,president,presAddr)`

`R2(title,year,studioName)`

R's FDs

Key

title year → studioName
studioName → president
president → presAddr

BCNF violations

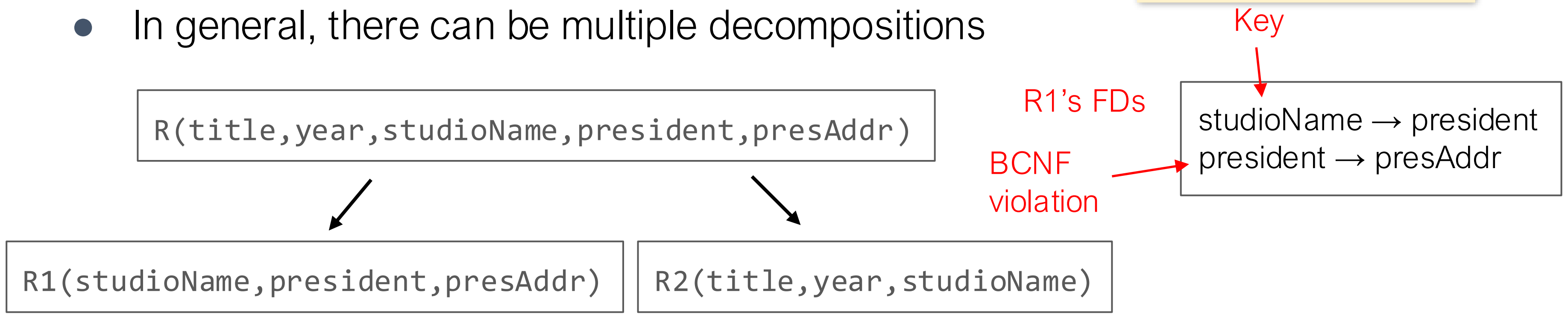
What is R1's FDs?

studioName → president
president → presAddr

Example: BCNF Decomposition

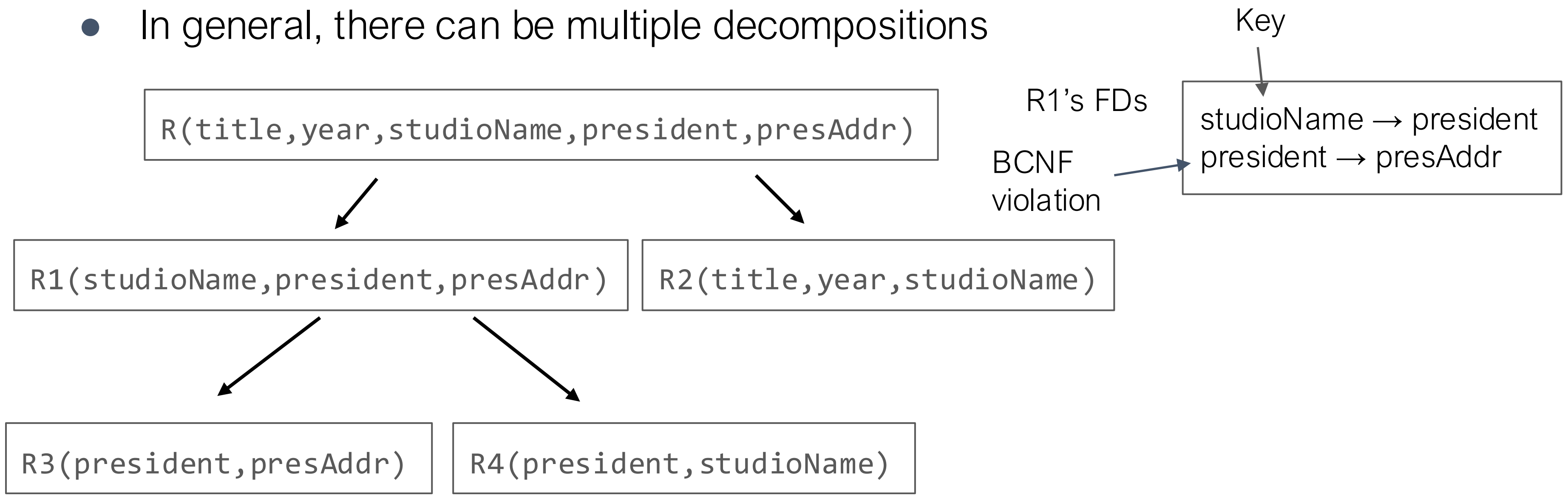
Is R1 in BCNF?

- In general, there can be multiple decompositions


R(title,year,studioName,president,presAddr)
R1(studioName,president,presAddr)
R2(title,year,studioName)
Key
R1’s FDs
studioName → president
president → presAddr
BCNF violation

Example: BCNF Decomposition

- In general, there can be multiple decompositions



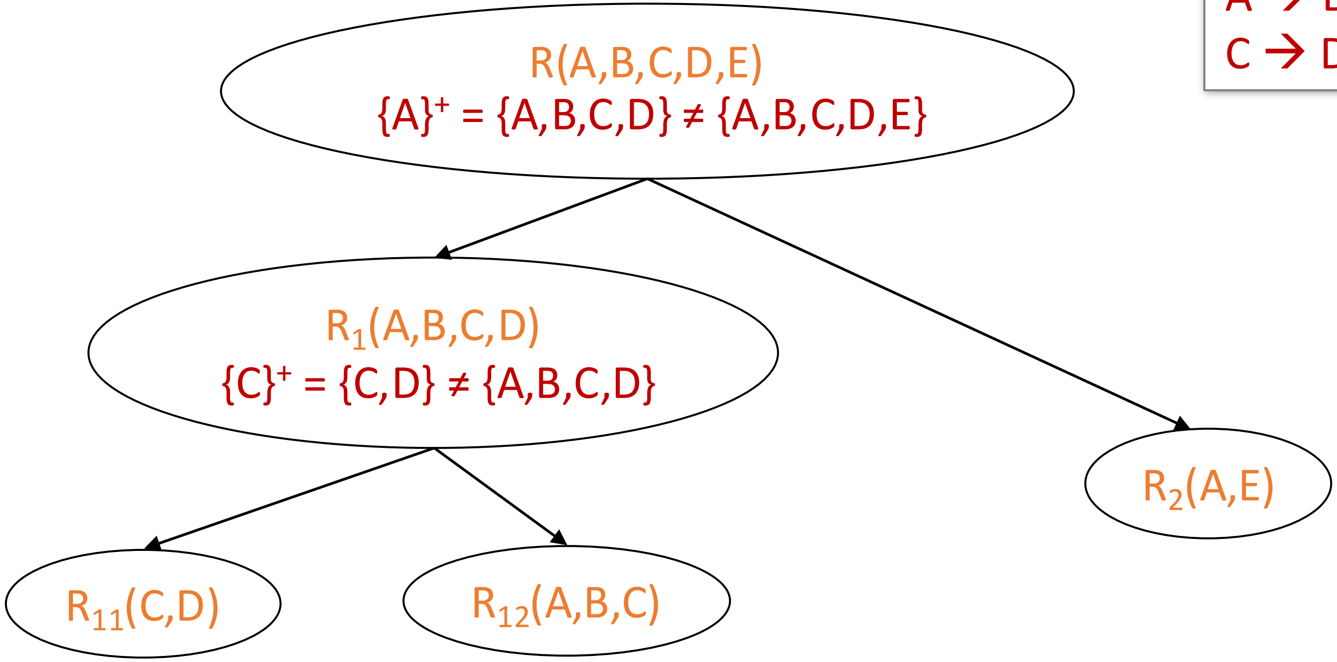
Q: Is this algorithm guaranteed to terminate successfully?

In-class Exercise

Decompose into relations satisfying BCNF

R(A,B,C,D,E)

$A \rightarrow BC$
$C \rightarrow D$



$R(A,B,C,D,E)$
$\{A\}^+ = \{A,B,C,D\} \neq \{A,B,C,D,E\}$
$R_1(A,B,C,D)$
$\{C\}^+ = \{C,D\} \neq \{A,B,C,D\}$
$R_2(A,E)$
$R_{11}(C,D)$
$R_{12}(A,B,C)$

2. Properties of Decomposition

Decompose to remove redundancies

1. We saw that **redundancies** in the data (“bad FDs”) can lead to data anomalies

2. We developed mechanisms to **detect and remove redundancies by decomposing tables into BCNF**

    1. BCNF decomposition is *standard practice* - very powerful & widely used!

3. However, sometimes decompositions can lead to **more subtle unwanted effects…**

When does this happen?

Recovering information from a decomposition

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

Name	Price
Gizmo	19.99
OneClick	24.99
~~Gizmo~~	~~19.99~~

Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

Sometimes a decomposition is "correct"

i.e. it is a Lossless decomposition

Recovering information from a decomposition

Name	Price	Category
Gizmo	19.99	Gadget
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Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

Price	Category
19.99	Gadget
24.99	Camera
19.99	Camera

However sometimes it isn't

What's wrong here?

Lossless Decompositions

$$\begin{tikzcd}
 & \boxed{R(A_1,\ldots,A_n,B_1,\ldots,B_m,C_1,\ldots,C_p)} \arrow[dl] \arrow[dr] & \\
\boxed{R_1(A_1,\ldots,A_n,B_1,\ldots,B_m)} & & \boxed{R_2(A_1,\ldots,A_n,C_1,\ldots,C_p)}
\end{tikzcd}$$

R_1 = the *projection* of R on $A_1, \ldots, A_n, B_1, \ldots, B_m$

R_2 = the *projection* of R on $A_1, \ldots, A_n, C_1, \ldots, C_p$

A decomposition R to (R1, R2) is lossless
if R = R1 Join R2

Lossless Decompositions

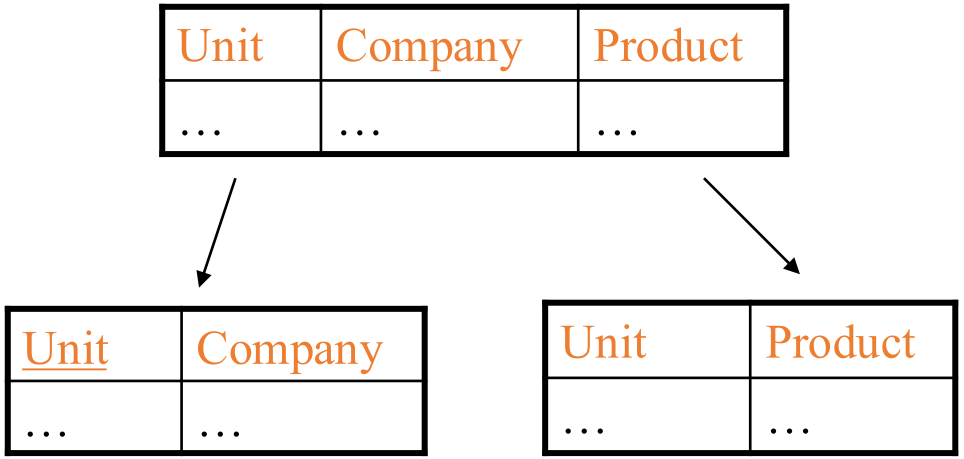
$$\begin{tikzcd}
 & \boxed{R(A_1,\ldots,A_n,B_1,\ldots,B_m,C_1,\ldots,C_p)} \arrow[dl] \arrow[dr] & \\
\boxed{R_1(A_1,\ldots,A_n,B_1,\ldots,B_m)} & & \boxed{R_2(A_1,\ldots,A_n,C_1,\ldots,C_p)}
\end{tikzcd}$$

If $A_1, \ldots, A_n \rightarrow B_1, \ldots, B_m$
Then the decomposition is lossless

Note: don't need
$A_1, \ldots, A_n \rightarrow C_1, \ldots, C_p$

BCNF decomposition is always lossless. Why?

A Problem with BCNF



Unit	Company	Product
…	…	…

Unit	Company
…	…

Unit	Product
…	…

Unit → Company

Unit → Company
Company,Product → Unit

We do a BCNF decomposition on a "bad" FD:
$\{\text{Unit}\}^{+} = \{\text{Unit, Company}\}$

We lose the FD Company,Product → Unit!!

So Why is that a Problem?

Unit	Company
Galaga99	UW
Bingo	UW

Unit	Product
Galaga99	Databases
Bingo	Databases

No problem so far. All local FD's are satisfied.

Unit → Company

Unit	Company	Product
Galaga99	UW	Databases
Bingo	UW	Databases

Let's put all the data back into a single table again:

Violates the FD Company,Product → Unit!!

The problem with BCNF

- We started with a table R and FDs F
- We decomposed R into BCNF tables R_1, R_2, …
  with their own FDs F_1, F_2, …
- We insert some tuples into each of the relations—which satisfy their local FDs but when reconstruct it violates some FD **across** tables!

Practical Problem: To enforce FD, must reconstruct R—on each insert!

Desirable properties of decomposition

(1) **Elimination of anomalies:** redundancy, update anomaly, delete anomaly

(2) **Recoverability of information**: can we recover the original relation by joining?

(3) **Preservation of dependencies**: if we check the projected FD's in the decomposed relations, does the reconstructed relation satisfy the original FD's

- BCNF gives (1) and (2), but not necessarily (3)
- 3NF gives (2) and (3), but not necessarily (1)
- In fact, there is no way to get all three at once!

3. 3NF

Third normal form (3NF)

A relation R is <u>in 3NF</u> if:

For every non-trivial FD $A_1, \ldots, A_n \rightarrow B$, either

- $\{A_1, \ldots, A_n\}$ is a superkey for R
- B is a prime attribute (i.e., B is part of some candidate key of R)

Example:

- The keys are AB and AC
- $B \rightarrow C$ is a BCNF violation, but not a 3NF violation because C is prime (part of the key AC)

R(A,B,C)

$AC \rightarrow B$
$B \rightarrow C$

3NF Decomposition Algorithm

3NFDecomp(R, F):

- Find minimal basis for F, say G
- For each FD X → A in G, if there is no relation that contains XA, create a new relation (X, A)
- Eliminate any relation that is a proper subset of another relation.
- If none of the resulting schemas are superkeys, add one more relation whose schema is a key for R

Keys:

ABE,ACE

Minimal basis:

$AB \rightarrow C$
$C \rightarrow B$
$A \rightarrow D$

R(A,B,C,D,E)

R_1(A,B,C)

R_2(B,C)

R_3(A,D)

R_4(A,B,E)

Exercise #2

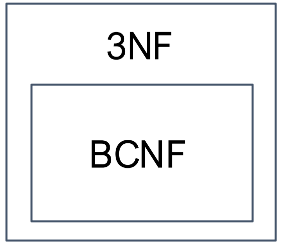
- What are the 3NF violations of the FDs?
- Decompose into relations satisfying 3NF

R(A, B, C, D)

$AB \rightarrow C$
$C \rightarrow D$
$D \rightarrow A$

BCNF vs 3NF

- Given a non-trivial FD $X \rightarrow B$ (X is a set of attributes)
  - BCNF: X must be a superkey
  - 3NF: X must be a superkey or B is prime
- Use 3NF over BCNF if you need dependency preservation
- However, 3NF may not remove all redundancies and anomalies



3NF relation:

A	B	C
1	2	3
3	2	3
2	3	1

F: $B \rightarrow C$, $AC \rightarrow B$

Can have redundancy and update anomalies

Can have deletion anomalies

4. MVDs

MVDs: Movie Star Example

Movie_ Star (A)	Address (B)	Movie (C)
Leonardo DiCaprio	Los Angeles	Titanic
Leonardo DiCaprio	Los Angeles	Inception
Leonardo DiCaprio	New York	Titanic
Leonardo DiCaprio	New York	Inception
Kate Winslet	Los Angeles	Titanic
Kate Winslet	Los Angeles	The Reader
Kate Winslet	London	Titanic
Kate Winslet	London	The Reader

Are there any functional dependencies that might hold here?

And yet it seems like there is some pattern / dependency…

MVDs: Movie Star Example

Movie_ Star (A)	Address (B)	Movie (C)
Leonardo DiCaprio	Los Angeles	Titanic
Leonardo DiCaprio	Los Angeles	Inception
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For a given movie star…

MVDs: Movie Star Example

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For a given movie star…

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Kate Winslet	Los Angeles	The Reader
Kate Winslet	London	Titanic
Kate Winslet	London	The Reader

For a given movie star…

Any address / movie combination is possible!

MVDs: Movie Star Example

	Movie_ Star (A)	Address (B)	Movie (C)
t_1	Leonardo DiCaprio	Los Angeles	Titanic
t_3	Leonardo DiCaprio	Los Angeles	Inception
	Leonardo DiCaprio	New York	Titanic
t_2	Leonardo DiCaprio	New York	Inception
	Kate Winslet	Los Angeles	Titanic
	Kate Winslet	Los Angeles	The Reader
	Kate Winslet	London	Titanic
	Kate Winslet	London	The Reader

More formally, we write $\{A\} \twoheadrightarrow \{B\}$ if for any tuples t_1,t_2 s.t. $t_1[A] = t_2[A]$, there is a tuple t_3 s.t.

- $t_3[A] = t_1[A]$

MVDs: Movie Star Example

	Movie_ Star (A)	Address (B)	Movie (C)
t_1	Leonardo DiCaprio	Los Angeles	Titanic
t_3	Leonardo DiCaprio	Los Angeles	Inception
	Leonardo DiCaprio	New York	Titanic
t_2	Leonardo DiCaprio	New York	Inception
	Kate Winslet	Los Angeles	Titanic
	Kate Winslet	Los Angeles	The Reader
	Kate Winslet	London	Titanic
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More formally, we write $\{A\} \twoheadrightarrow \{B\}$ if for any tuples t_1,t_2 s.t. $t_1[A] = t_2[A]$, there is a tuple t_3 s.t.

- $t_3[A] = t_1[A]$
- $t_3[B] = t_1[B]$

MVDs: Movie Star Example

	Movie_ Star (A)	Address (B)	Movie (C)
t_1	Leonardo DiCaprio	Los Angeles	Titanic
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	Kate Winslet	Los Angeles	Titanic
	Kate Winslet	Los Angeles	The Reader
	Kate Winslet	London	Titanic
	Kate Winslet	London	The Reader

More formally, we write $\{A\} \twoheadrightarrow \{B\}$ if for every pair of tuples t_1,t_2 s.t. $t_1[A] = t_2[A]$, there exisits a tuple t_3 s.t.

- $t_3[A] = t_1[A]$
- $t_3[B] = t_1[B]$
- and $t_3[R\backslash B] = t_2[R\backslash B]$

Where R\B is "R minus B" i.e. the attributes of R not in B

MVDs: Movie Star Example

	Movie_ Star (A)	Address (B)	Movie (C)
t_2	Leonardo DiCaprio	Los Angeles	Titanic
	Leonardo DiCaprio	Los Angeles	Inception
t_3	Leonardo DiCaprio	New York	Titanic
t_1	Leonardo DiCaprio	New York	Inception
	Kate Winslet	Los Angeles	Titanic
	Kate Winslet	Los Angeles	The Reader
	Kate Winslet	London	Titanic
	Kate Winslet	London	The Reader

Note this also works!

An MVD holds over a relation or an instance, so defn. must hold for *every applicable pair*…

*There are no restrictions on t1, t2, t3. They can be the same or different.

MVDs: Movie Star Example

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This expresses a sort of dependency (= data redundancy) that we can't express with FDs

*Actually, it expresses conditional independence (between address and movie given movie star)!

Multi-Value Dependencies (MVDs)

A multi-value dependency (MVD) is another type of dependency that could hold in our data, ***which is not captured by FDs***

- Every FD is an MVD

Definition:

- Given a relation R, attribute set A, and two sets of attributes $\boldsymbol{X}, \boldsymbol{Y} \subseteq \boldsymbol{A}$
- The multi-value dependency (MVD) $\boldsymbol{X} \twoheadrightarrow \boldsymbol{Y}$ holds on R if for any tuples $\boldsymbol{t_1}, \boldsymbol{t_2} \in \boldsymbol{R}$ s.t. $\boldsymbol{t_1}[\boldsymbol{X}] = \boldsymbol{t_2}[\boldsymbol{X}]$, there exists a tuple $\boldsymbol{t}_3$ s.t.:
  - $t_1[X] = t_2[X] = t_3[X]$
  - $t_1[Y] = t_3[Y]$
  - $t_2[A \backslash Y] = t_3[A \backslash Y]$

A \ B means “elements of set A not in set B”

Multi-Value Dependencies (MVDs)

One less formal, literal way to phrase the definition of an MVD:

The MVD $\boldsymbol{X} \twoheadrightarrow \boldsymbol{Y}$ holds on R if for any pair of tuples with the same X values, the tuples with the same X values, but the other permutations of Y and A\Y values, is also in R

Ex: X = {x}, Y = {y}:

x	y	z
1	0	1
1	1	0

For $\boldsymbol{X} \twoheadrightarrow \boldsymbol{Y}$ to hold must have…

x	y	z
1	0	1
1	1	0
1	0	0
1	1	1

Multi-Value Dependencies (MVDs)

Another way to understand MVDs, in terms of conditional independence:

The MVD $\mathbf{X} \twoheadrightarrow \boldsymbol{Y}$ holds on R if given X, Y is conditionally independent of A \ Y and vice versa…

Here, given x = 1, we know for ex. that:
y = 0 → z = 1

I.e. z is conditionally *dependent* on y given x

x	y	z
1	0	1
1	1	0

Here, this is not the case!

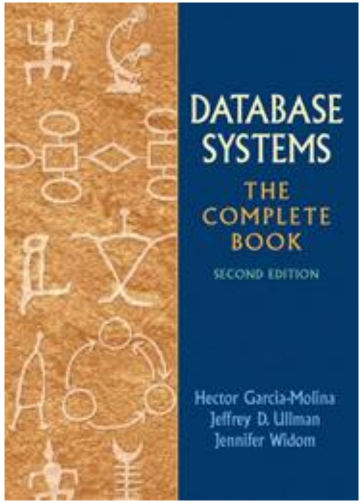
I.e. z is conditionally *independent* of y given x

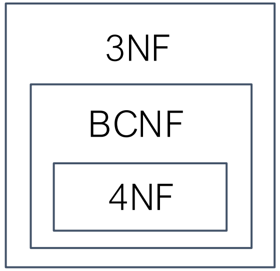
x	y	z
1	0	1
1	1	0
1	0	0
1	1	1

Further Readings (Chapter 3.6)

4NF: Remove MVD redundancies

Property	3NF	BCNF	4NF
Lossless join	Y	Y	Y
Eliminates FD redundancies	N	Y	Y
Eliminates MVD redundancies	N	N	Y
Preserves FD's	Y	N	N
Preserves MVD's	N	N	N


DATABASE SYSTEMS
THE COMPLETE BOOK
SECOND EDITION
Hector Garcia-Molina
Jeffrey D. Ullman
Jennifer Widom



Summary

Good schema design is important

- Avoid redundancy and anomalies
- Functional dependencies

Normal forms describe how to **remove** this redundancy by **decomposing** relations

- BCNF gives elimination of anomalies and lossless join
- 3NF gives lossless join and dependency preservation

BCNF is intuitive and most widely used in practice