CS 6400 A

# Database Systems Concepts and Design

Lecture 9
09/18/24

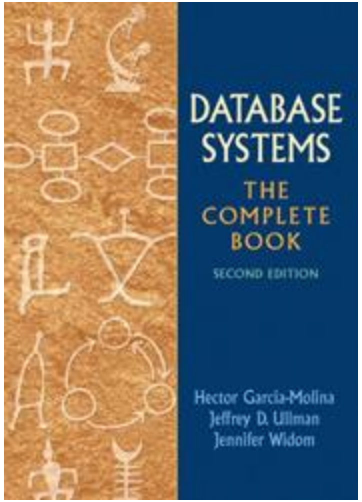
# Agenda

1. B+-Tree cost model
2. Hashing

# Reading Materials

Database Systems: The Complete Book (2nd edition)

- Chapter 14.3: Hash Tables




Acknowledgement: The following slides have been adapted from EE477 (Database and Big Data Systems) taught by Steven Whang and CS145 (Intro to Big Data Systems) taught by Peter Bailis.

# 1. B+-Tree cost model

# B+ Tree: High Fanout = Smaller & Lower IO

So why does B+ tree work?

As compared to binary search trees, B+ Trees have **high *fanout*** (*between d+1 and 2d+1*)

This means that the **depth of the tree is small →** getting to any element requires very few IO operations!

- Also can often store most or all of the B+ Tree in main memory!

The fanout is defined as the number of pointers to child nodes coming out of a node

Note that fanout is dynamic - we'll often assume it's constant just to come up with approximate eqns!

# B+ Trees in Practice

Typical order: d=100. Typical fill-factor: 67%.

- average fanout = 133

Top levels of tree sit *in the buffer pool*:

- Level 1 = 1 page = 8 KB
- Level 2 = 133 pages = 1 MB
- Level 3 = 17,689 pages = 133 MB

Fill-factor is the percent of available slots in the B+ Tree that are filled; is usually < 1 to leave slack for (quicker) insertions

Typically, only pay for **one IO**!

# Simple Cost Model for Search

Suppose:

- $f$ = fanout, which is in [d+1, 2d+1] *(we'll assume it's constant for our cost model…)*
- $N$ = the total number of *pages* we need to index
- $F$ = fill-factor (usually ~= 2/3)

Our B+ Tree needs to have room to index $N / F$ pages!

- We have the fill factor in order to leave some open slots for faster insertions

What height ($h$) does our B+ Tree need to be?

- h=1 → Just the root node- room to index f pages
- h=2 → f leaf nodes- room to index $f^2$ pages
- h=3 → $f^2$ leaf nodes- room to index $f^3$ pages
- …
- h → $f^{h-1}$ leaf nodes- room to index $f^h$ pages!

→ We need a B+ Tree of height $h = \left\lceil \log_f \frac{N}{F} \right\rceil$!

# Simple Cost Model for Search

Note that if we have *B* available buffer pages, by the same logic:

- We can store $\boldsymbol{L_B}$ levels of the B+ Tree in memory
- where $\boldsymbol{L_B}$ *is the number of levels such that the sum of all the levels' nodes fit in the buffer:*
    - $B \geq 1 + f + \cdots + f^{L_B - 1} = \sum_{l=0}^{L_B - 1} f^l$

In summary: to do exact search:

- We read in one page per level of the tree
- However, levels that we can fit in buffer are free!
- Finally we read in the actual record

IO Cost: $\left\lceil \log_f \frac{N}{F} \right\rceil - L_B + 1$

*where* $B \geq \sum_{l=0}^{L_B - 1} f^l$

# Simple Cost Model for Search

To do range search, we just follow the horizontal pointers

The IO cost is that of loading additional leaf nodes we need to access + the IO cost of loading each *page* of the results- we phrase this as “Cost(OUT)”

IO Cost: $\left\lceil \log_f \frac{N}{F} \right\rceil - L_B + Cost(OUT)$

*where* $B \geq \sum_{l=0}^{L_B - 1} f^l$

# 2. Hashing

# Indexing vs hashing

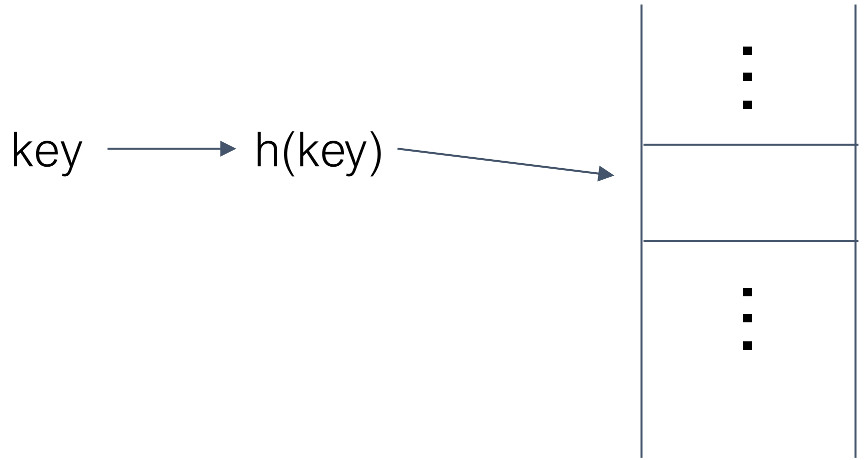
- Indexing (including B+ trees) is good for range lookups
- Hashing is good for equality-based point lookups

```
SELECT *
FROM Movies
WHERE year >= 2000;
```

```
SELECT *
FROM Movies
WHERE title = ‘Ponyo’;
```

# Hash table

- A hash function h takes a key and returns a block number from 0 to B - 1
- Blocks contain records and are stored in secondary storage
- Complexity:
  - O(1) operation complexity
  - O(n) storage complexity



# Hash table: Design Decisions

Hash Function

- How to map a large key space into a smaller domain of array offsets
- Trade-off between fast execution vs. collision rate

Hashing Scheme

- How to handle key collisions after hashing
- Trade-off between allocating a large hash table vs. extra steps to location/insert keys

# Hash function

- For any input key, return an integer representation of that key.
  - Output is deterministic
- Example:
  - Given a key that is a string, return the sum of the characters $x_i$ modulo B (i.e., $\Sigma x_i$ % B)
  - This function is not idea since there might be many collisions
- We do NOT want to use a cryptographic hash function (e.g., SHA-256) for DBMS hash tables
- In general, we only care about the hash function’s speed and collision rate.
- Current SOTA: xxHash

# Static hash table

- The number of buckets is fixed
- Often used during query execution because they are faster than dynamic hashing schemes.
- If the DBMS runs out of storage space in the hash table, it has to rebuild a larger hash table (usually 2x) from scratch, which is very expensive!

Examples

- Linear Probing Hashing
- Robinhood Hashing (not covered)
- Cuckoo Hashing

# Linear Probing Hashing

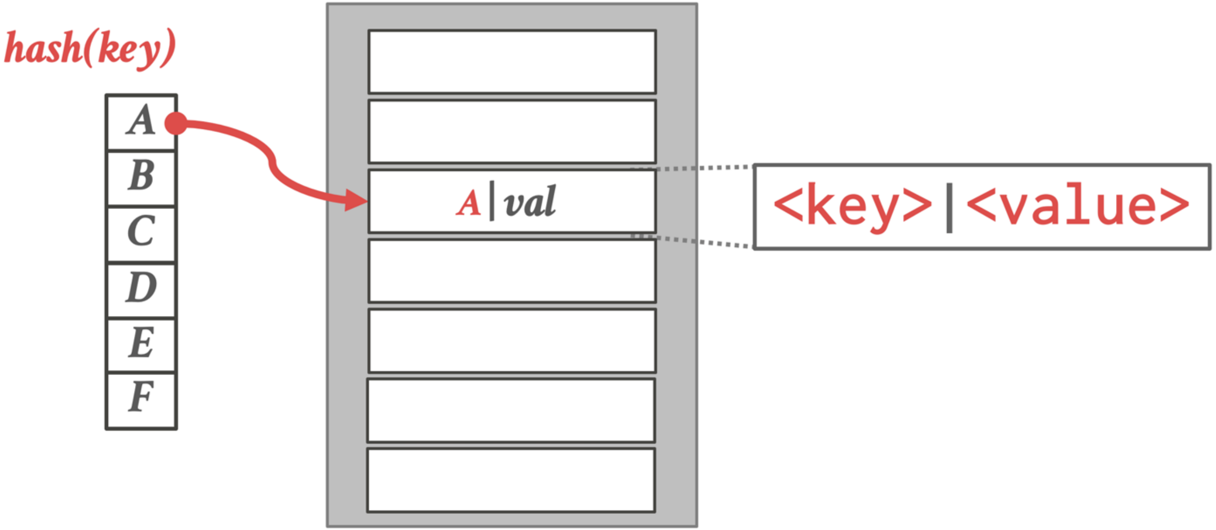
Single giant table of slots

Resolve collisions by linearly searching for the next free slot in the table.

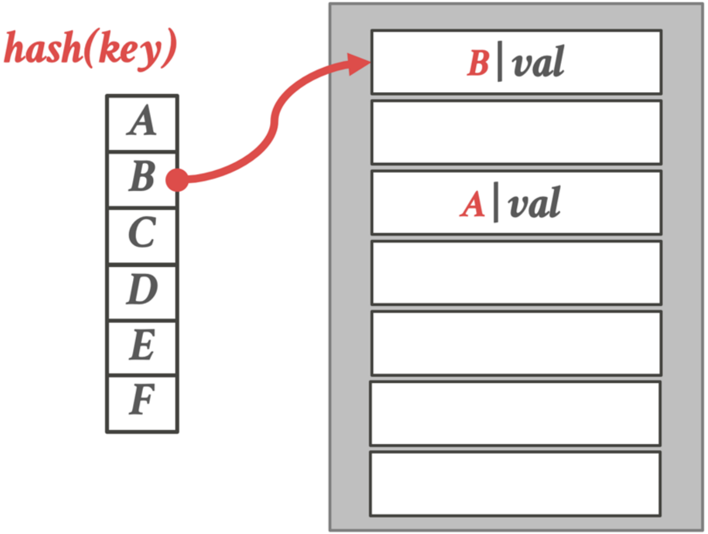
- To determine whether an element is present, hash to a location in the index and scan for it.
- Has to store the key in the index to know when to stop scanning
- Insertions and deletions are generalizations of lookups

Example: Google's absl::flat_hash_map

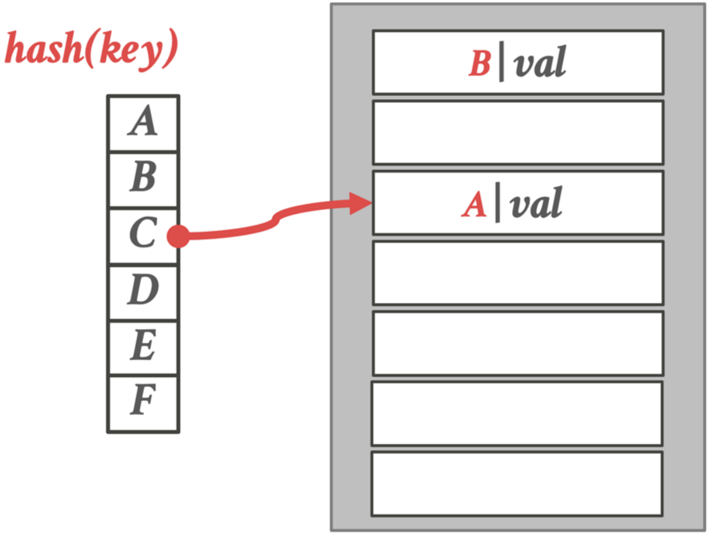
# Linear Probing Hashing



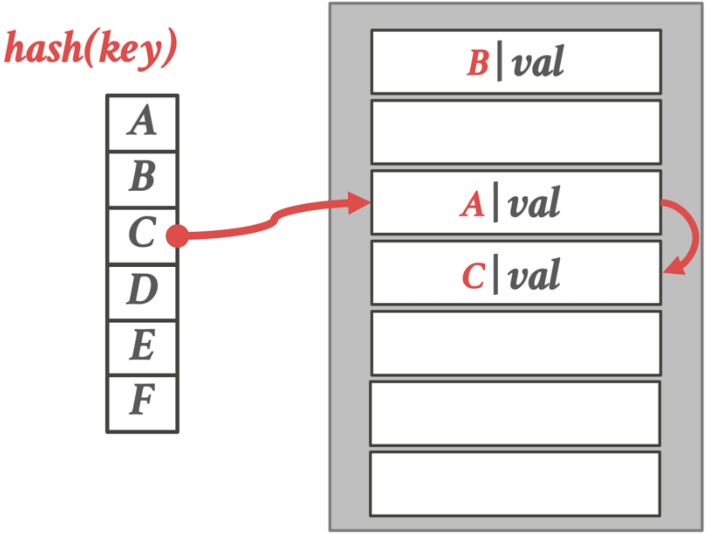
# Linear Probing Hashing



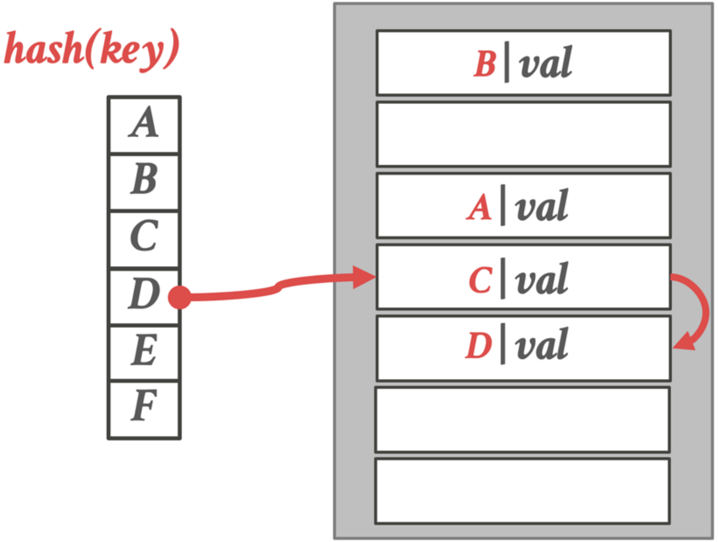
# Linear Probing Hashing



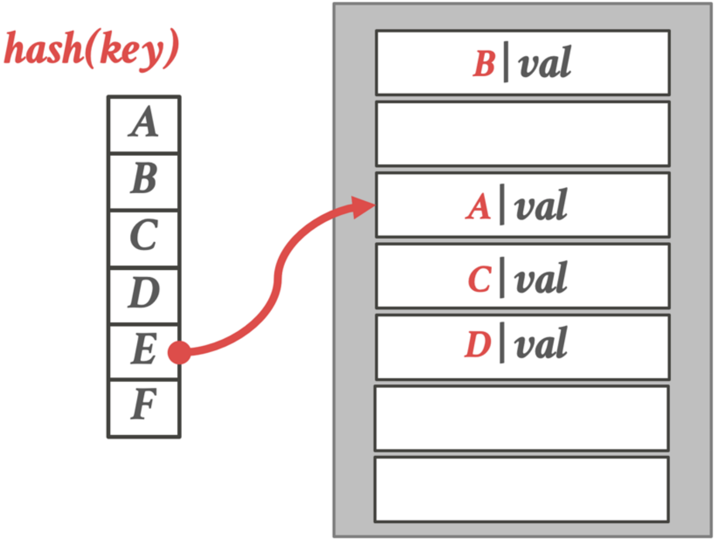
# Linear Probing Hashing



# Linear Probing Hashing

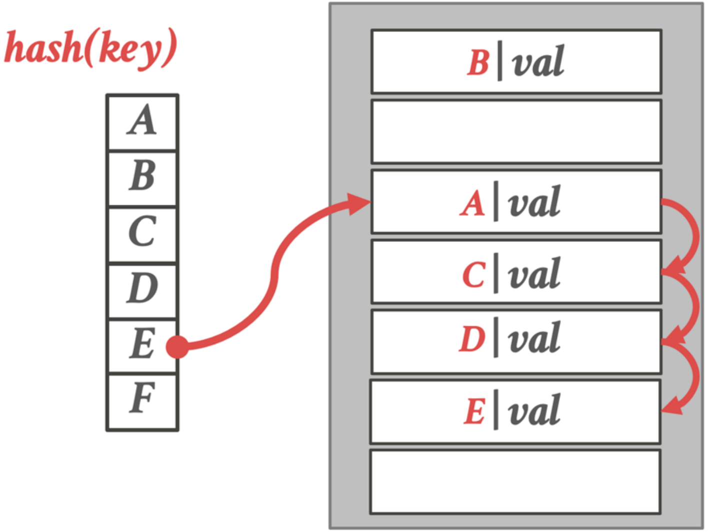


# Linear Probing Hashing

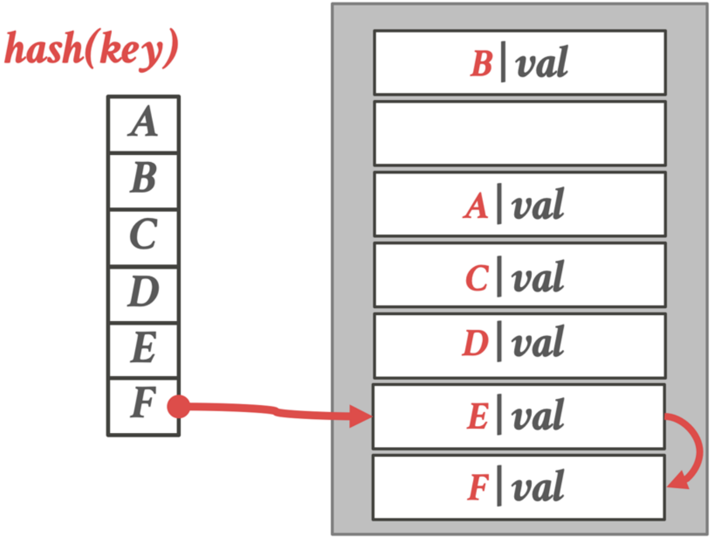



Q: What would happen in this case?

# Linear Probing Hashing



# Linear Probing Hashing



# Linear Probing Hashing - Delete

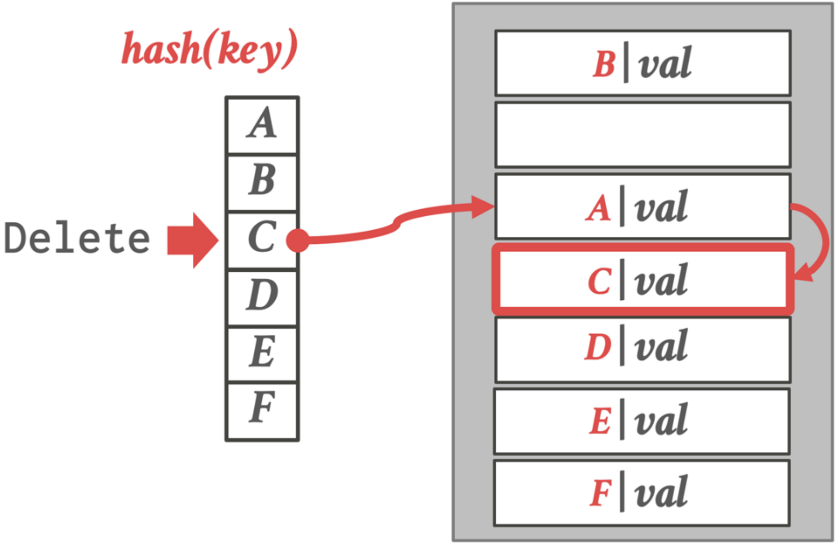
It is not sufficient to simply delete the key

This would affect searches for keys that have a hash value earlier than the emptied cell, but are stored in a position later than the emptied cell.

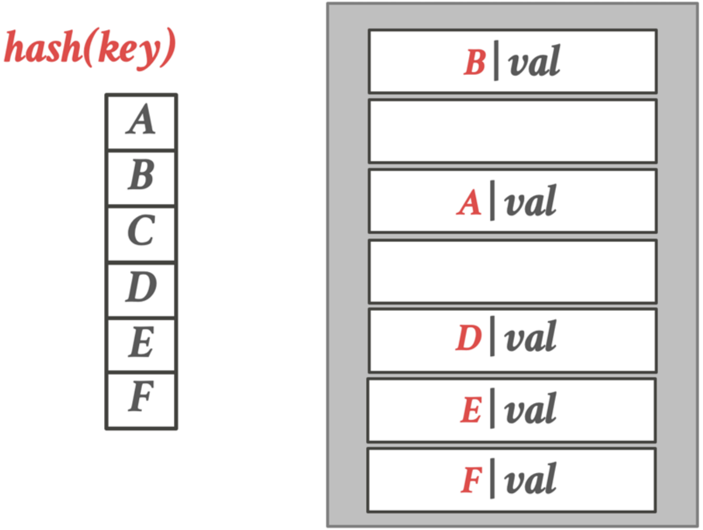
Two solutions:

- Tombstone
- Movement (less common)

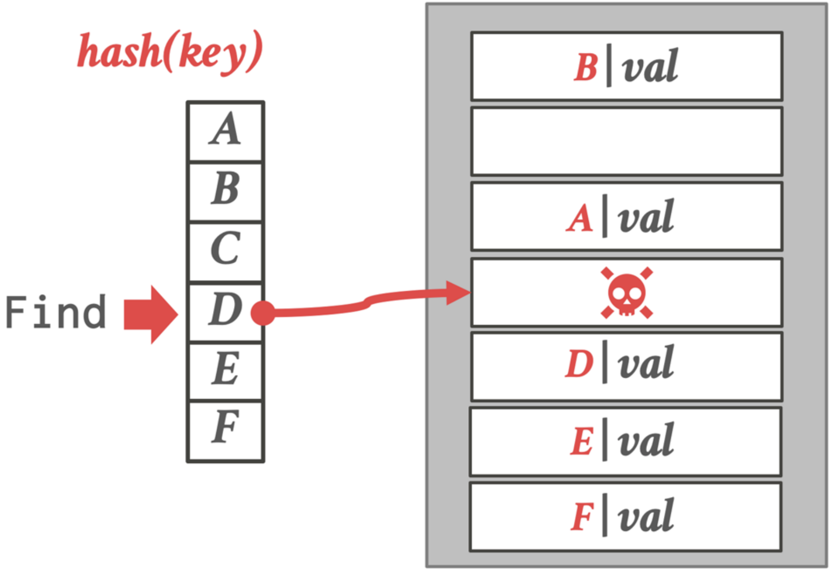
# Linear Probing Hashing



# Linear Probing Hashing

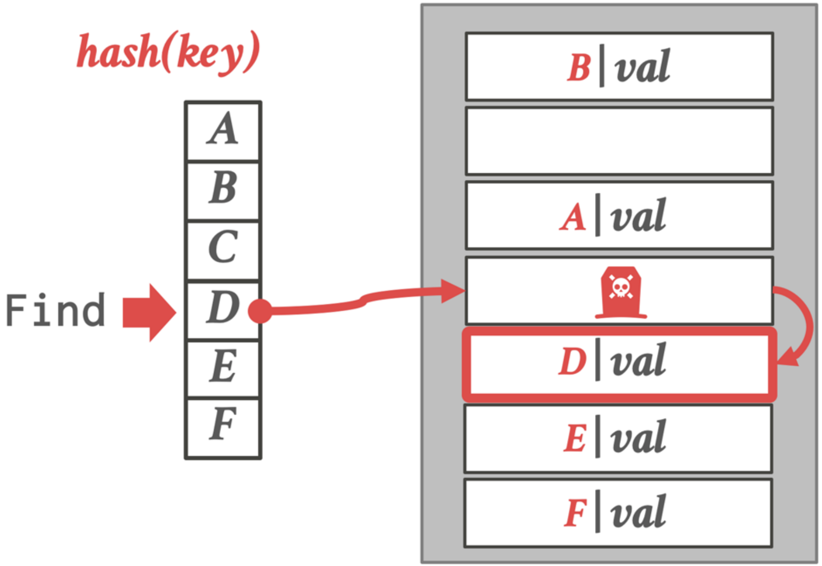


# Linear Probing Hashing



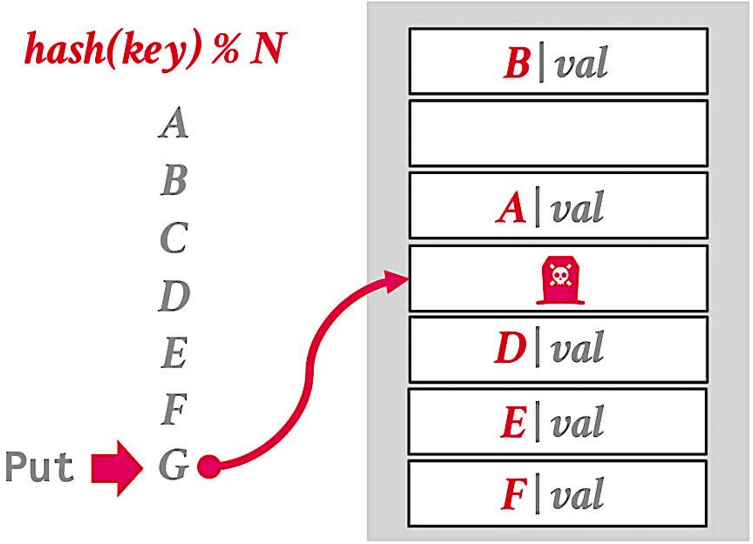

- Set a marker to indicate that the entry in the slot is logically deleted.

# Linear Probing Hashing



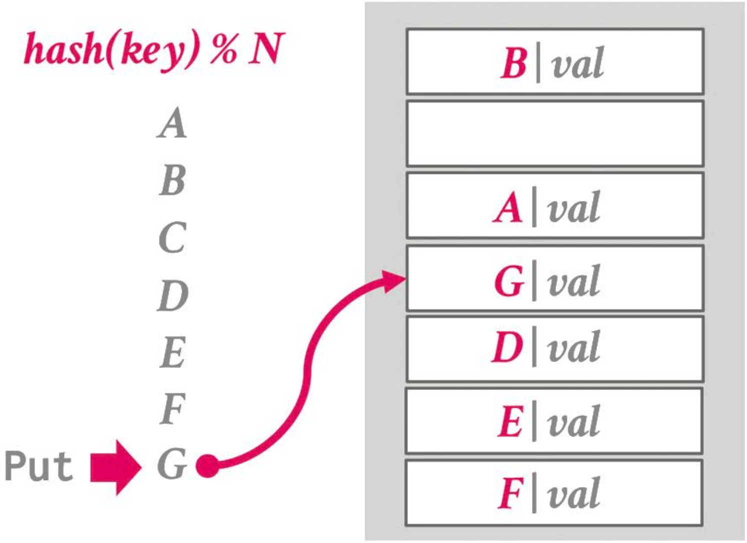

- Set a marker to indicate that the entry in the slot is logically deleted.

# Linear Probing Hashing




- Set a marker to indicate that the entry in the slot is logically deleted.
- Reuse the slot for new keys

# Linear Probing Hashing




- Set a marker to indicate that the entry in the slot is logically deleted.
- Reuse the slot for new keys

# Cuckoo Hashing

Power of 2 choices: Use multiple hash tables with different seeds

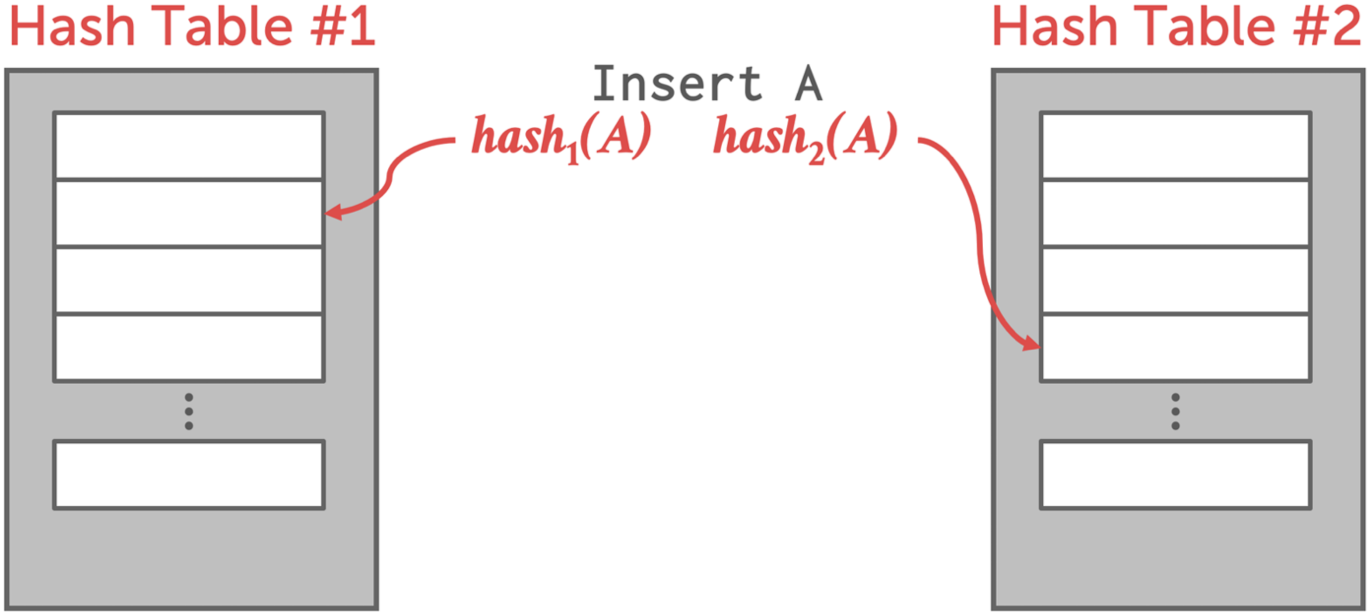
- On insert, check every table and pick one with a free slot
- If no table has a free slot, evict the element from one of then and then re-hash it to find a new location
- In rare cases, we may end up in a cycle. If this happens, we can rebuild using larger hash tables

Look-ups and deletions are ~O(1) because only one location per hash table is checked.

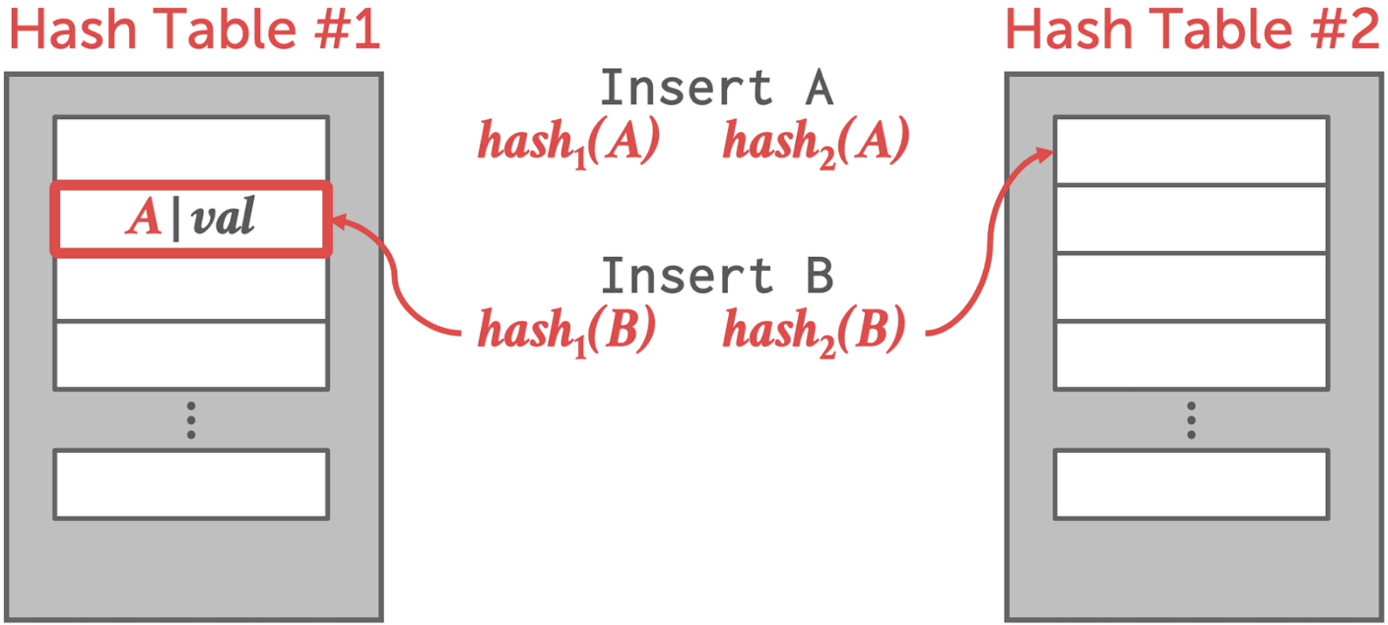


Image source: https://theconversation.com/egg-colours-make-cuckoos-masters-of-disguise-34217

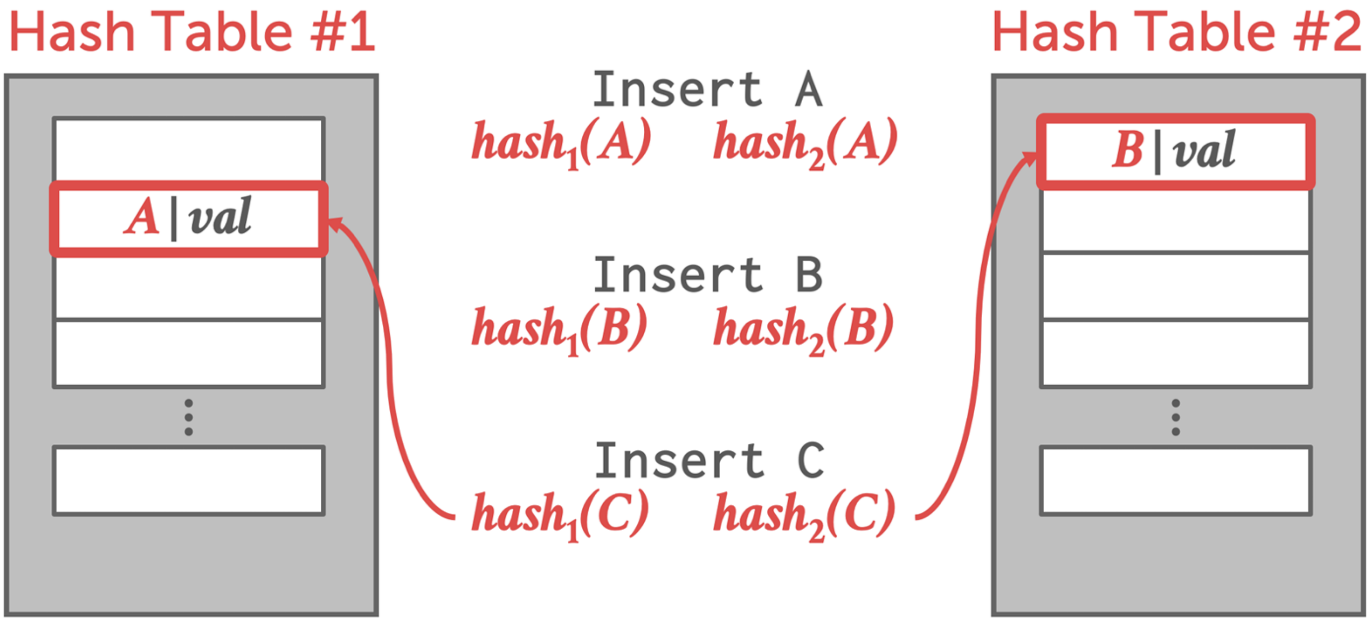
# Cuckoo Hashing



# Cuckoo Hashing



# Cuckoo Hashing



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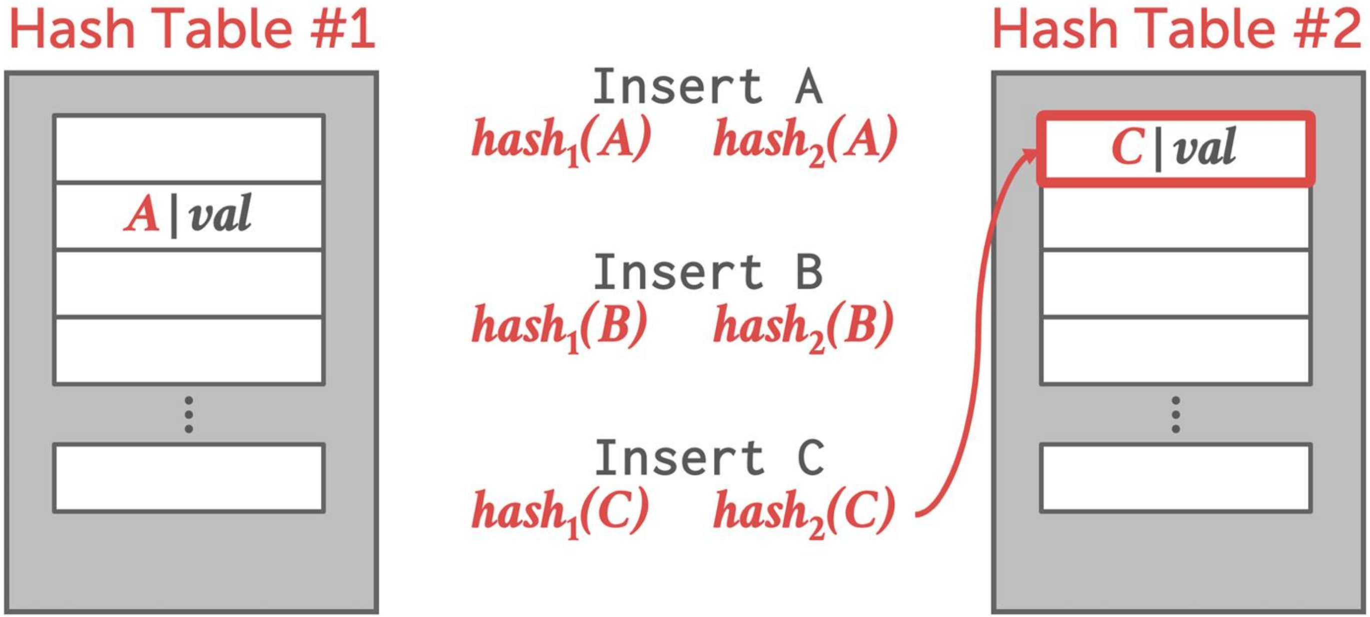
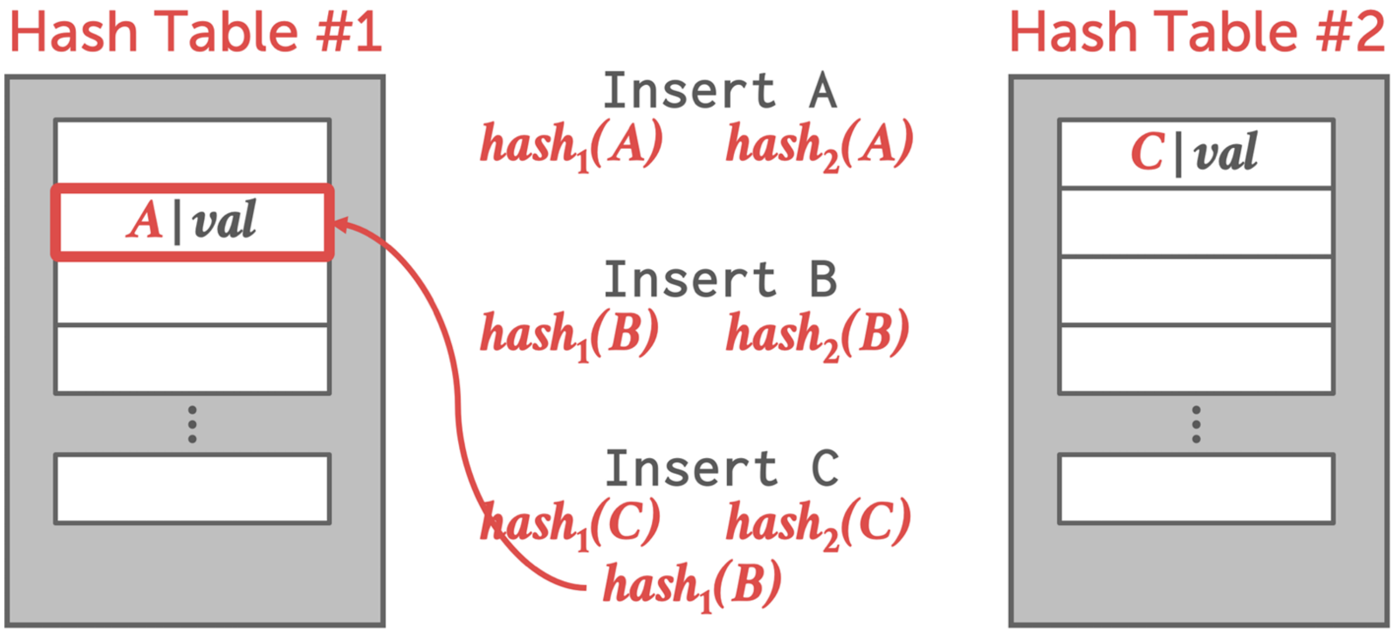
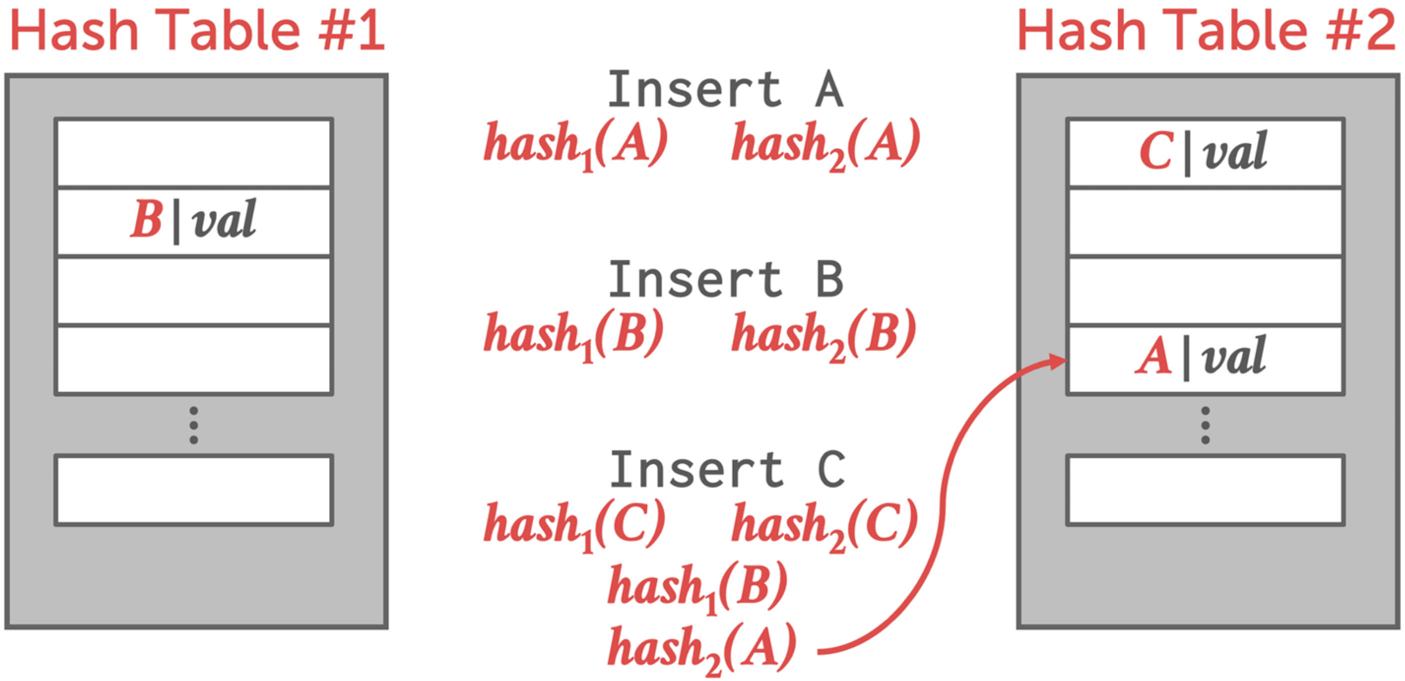


# Cuckoo Hashing



# Cuckoo Hashing



# Dynamic hash table

- The previous hash tables require the DBMS to know the number of elements it wants to store.
  - Otherwise it needs to rebuild the table to resize
- Dynamic hash tables incrementally resize the hash table on demand without needing to rebuild the entire table.

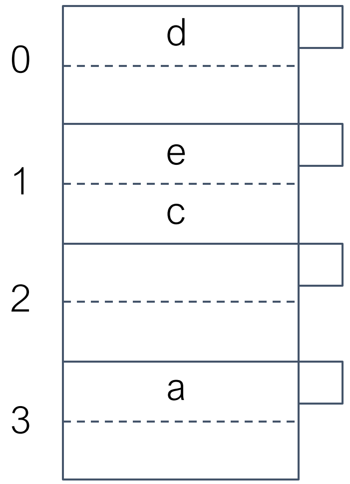
Examples

- Chained Hashing
- Extensible Hashing
- Linear Hashing

# Chained Hashing

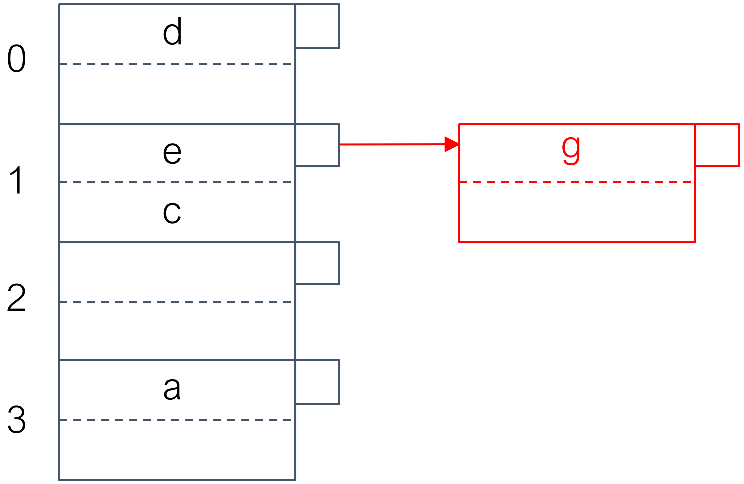
- Maintain a linked list of buckets for each slot in the hash table.

- Resolve collisions by placing all elements with the same hash key into the same bucket.
  - To determine whether an element is present, hash to its bucket and scan for it.
  - Insertions and deletions are generalizations of lookups.



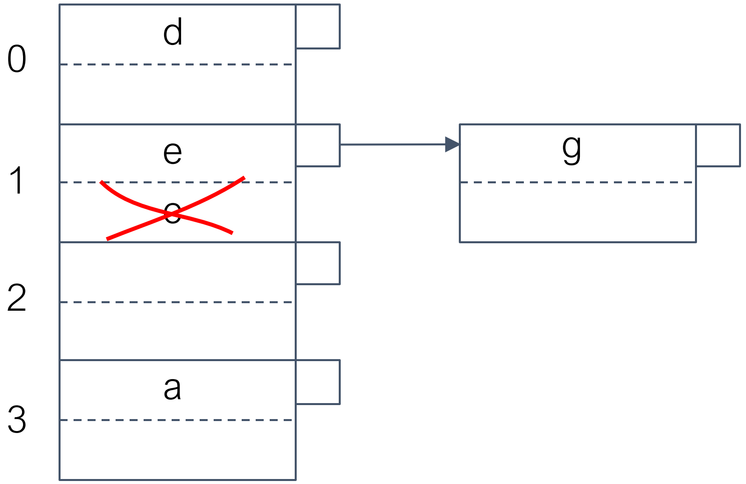
# Chained Hashing

- Add g where h(g) = 1



# Chained Hashing

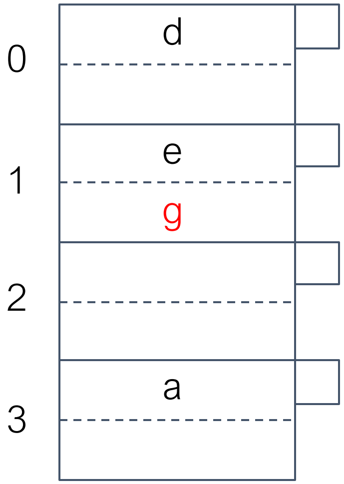
- Remove c where h(c) = 1



# Chained Hashing

- Remove c where h(c) = 1

Q: What can go wrong with chained hashing?



# Extendible Hashing

Chained-hashing approach that splits buckets incrementally instead of letting the linked list grow forever.

- Long chains of blocks -> many disk I/Os

Multiple slot locations can point to the same bucket chain.

Reshuffle bucket entries on split and increase the number of bits to examine.

- Data movement is localized to just the split chain.

# Extensible hash table

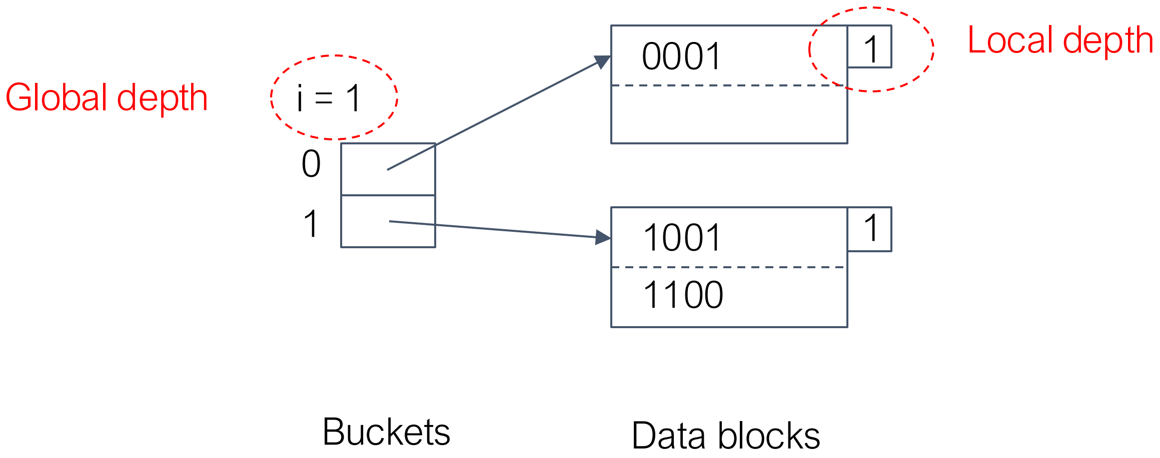
- Use first i bits of hash value to locate block
  - i grows over time

i = 3

h(key):
00101100

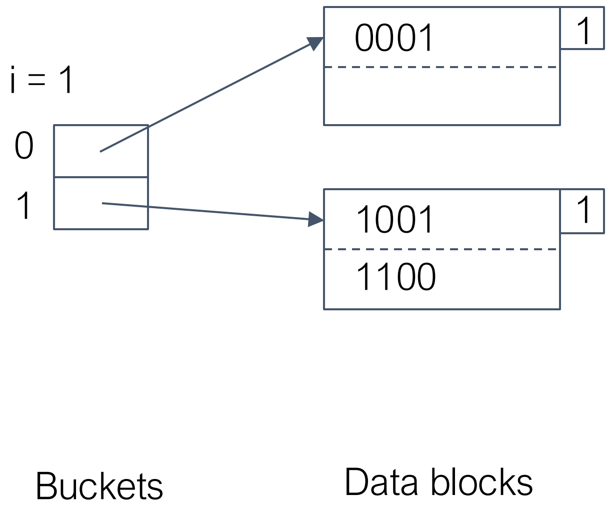
# Extensible hash table

- Use level of indirection where buckets are pointers to blocks



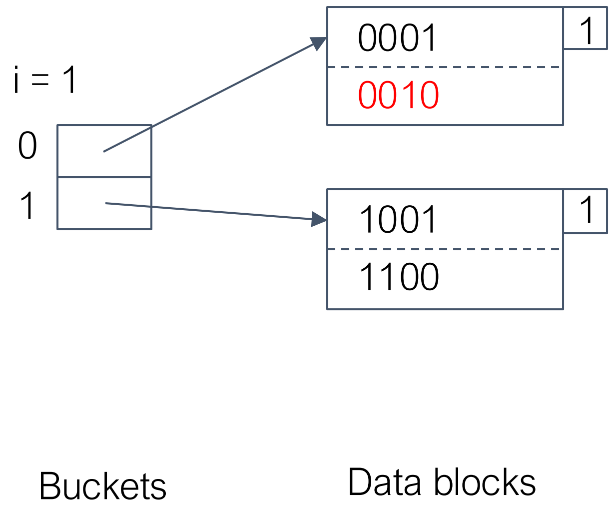
# Extensible hash table

- Add 0010



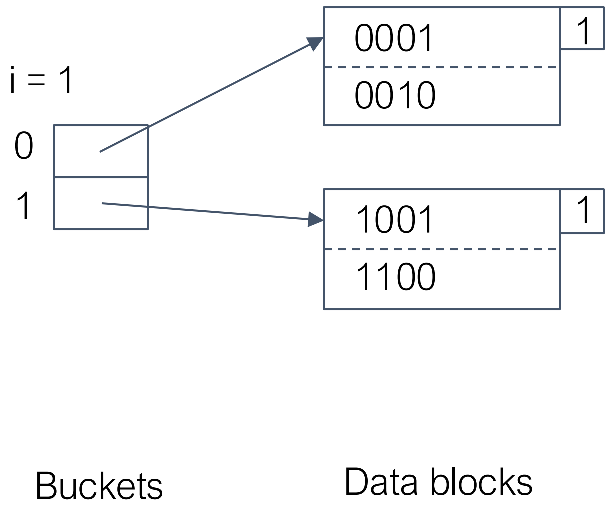
# Extensible hash table

- Add 0010



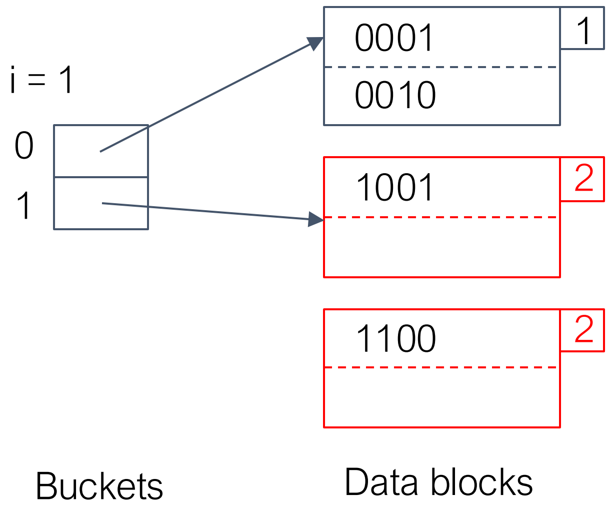
# Extensible hash table

- Add 1010



# Extensible hash table

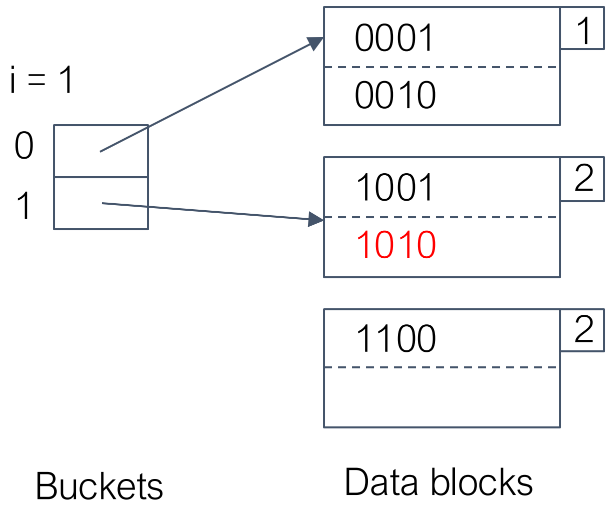
- Add 1010




May need to repeat splitting until there is space

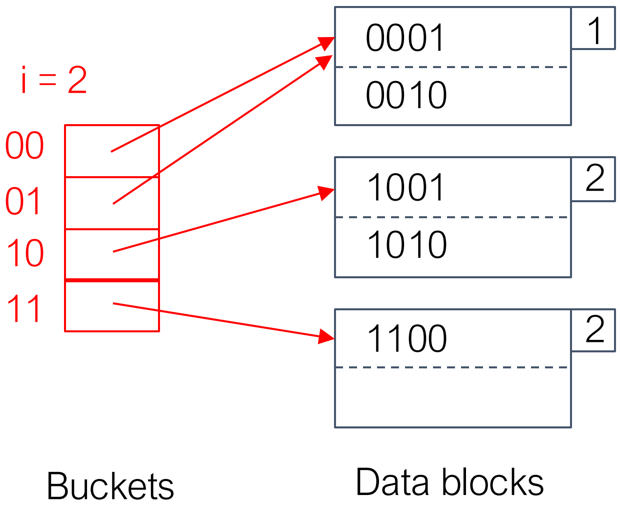
# Extensible hash table

- Add 1010



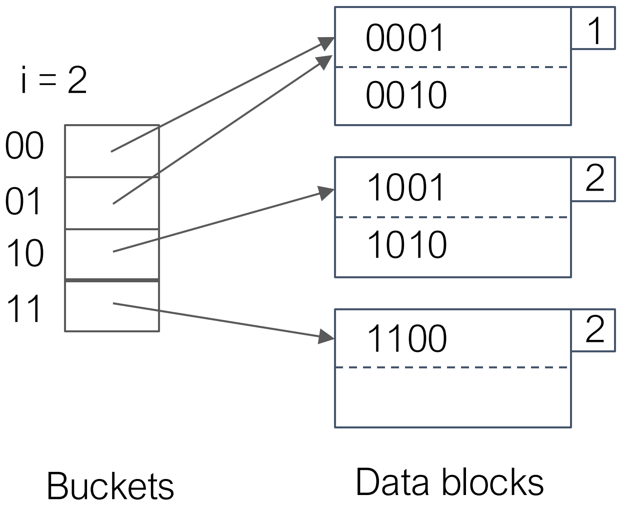
# Extensible hash table

- Add 1010



# Extensible hash table

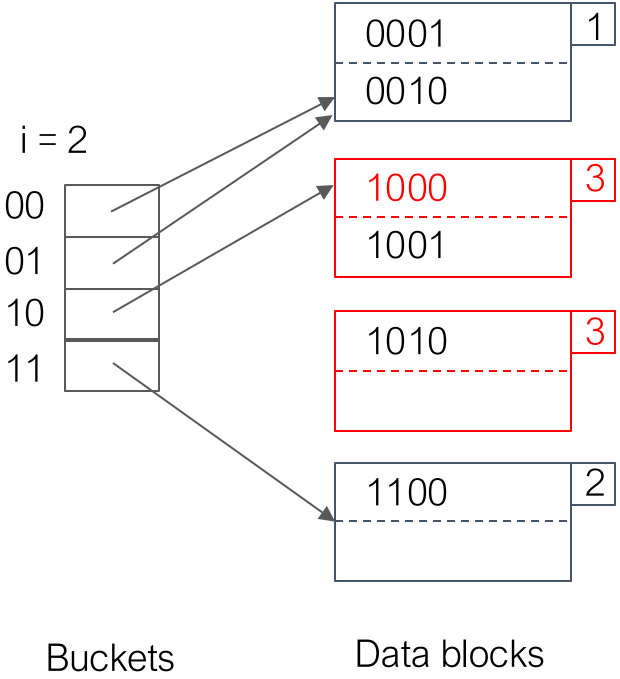
- Add 1000




Q: What will happen in this case?

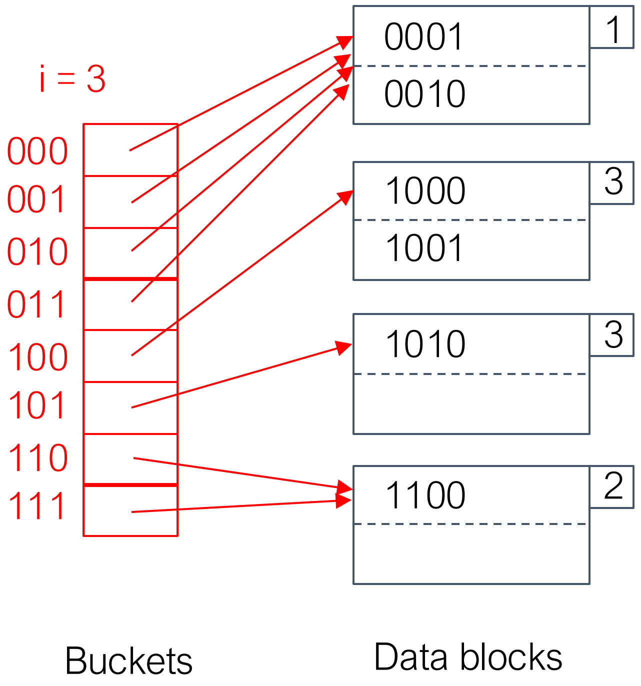
# Extensible hash table

- Add 1000



# Extensible hash table

- Add 1000



# Extensible hashing summary

If bucket array fits in memory, lookup is always 1 disk I/O

Can grow table with little wasted space and avoiding full reorganizations

However, doubling the bucket array is expensive

- Splitting can occur frequently if the number of records per block is small
- At some point, the bucket array may not fit in memory

Linear hashing (covered next) grows the number of buckets more slowly

# Linear hashing

The hash table maintains a pointer that tracks the next bucket to split.

- When any bucket overflows, split the bucket at the pointer location.

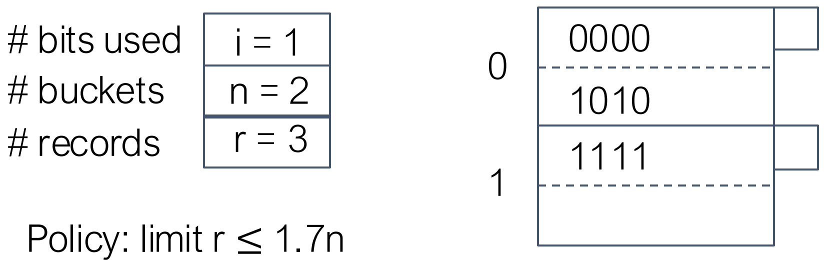
Use multiple hashes to find the right bucket for a given key.

Can use different overflow criterion:

- Space Utilization
- Average Length of Overflow Chains

# Linear hash tables

- Use last i bits of hash value to locate block
- Hash table grows linearly



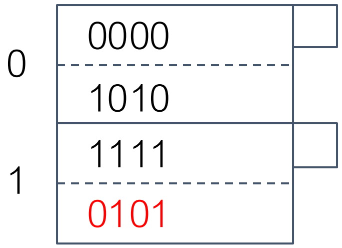
# Linear hash tables

- Add 0101

| | |
|---|---|
| # bits used | i = 1 |
| # buckets | n = 2 |
| # records | r = 4 |

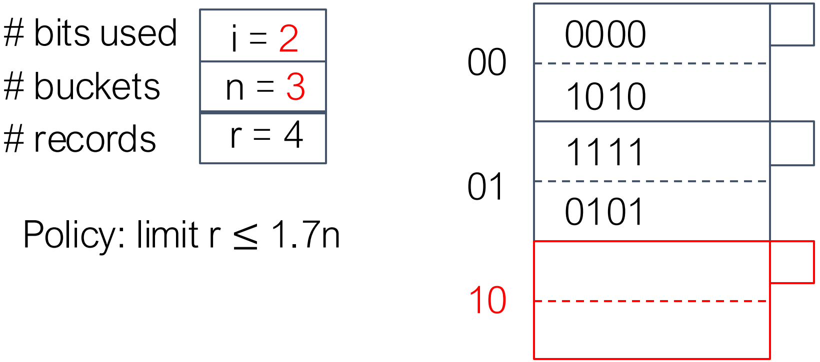
Policy: limit r ≤ 1.7n

Violation



# Linear hash tables

- Add 0101

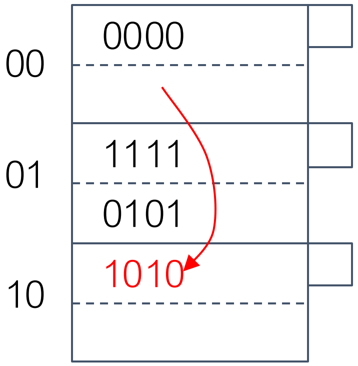


# Linear hash tables

- Add 0101

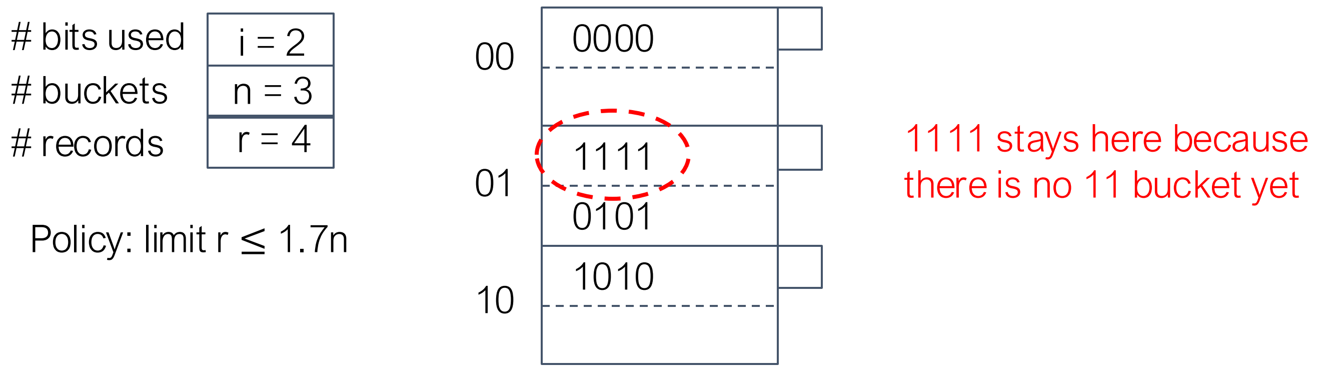
| | |
|---|---|
| # bits used | i = 2 |
| # buckets | n = 3 |
| # records | r = 4 |

Policy: limit r ≤ 1.7n



# Linear hash tables

- Add 0101

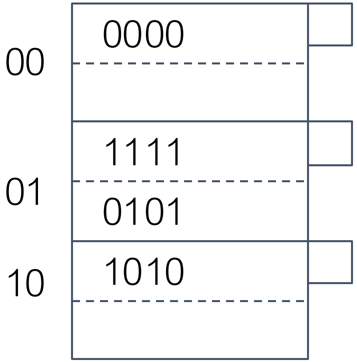


# Linear hash tables

- Add 0001

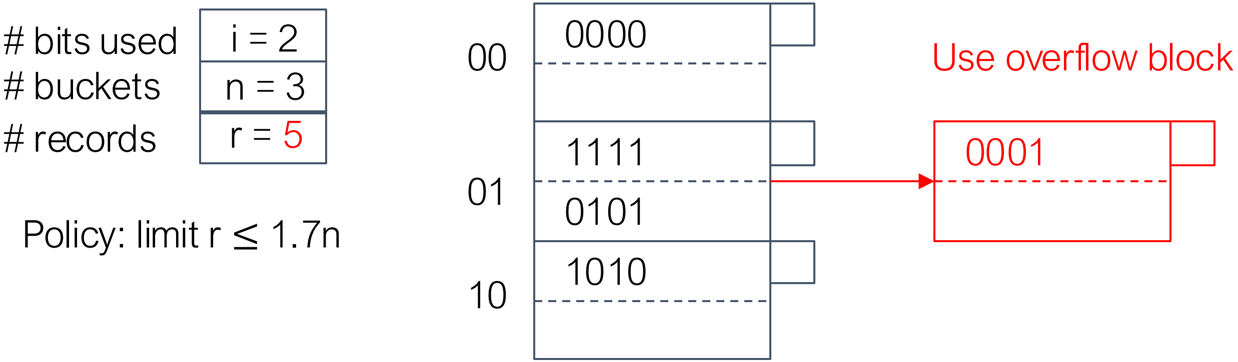
| | |
|---|---|
| # bits used | i = 2 |
| # buckets | n = 3 |
| # records | r = 4 |

Policy: limit $r \leq 1.7n$



# Linear hash tables

- Add 0001



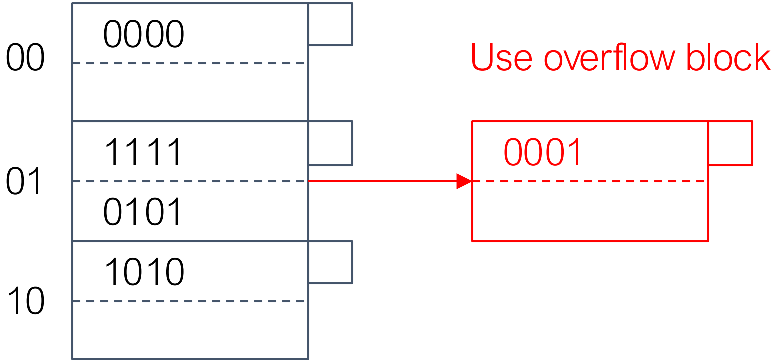
# Linear hash tables

- Add 0001

| | |
|---|---|
| # bits used | i = 2 |
| # buckets | n = 3 |
| # records | r = 5 |

Policy: limit r ≤ 1.7n

No violation

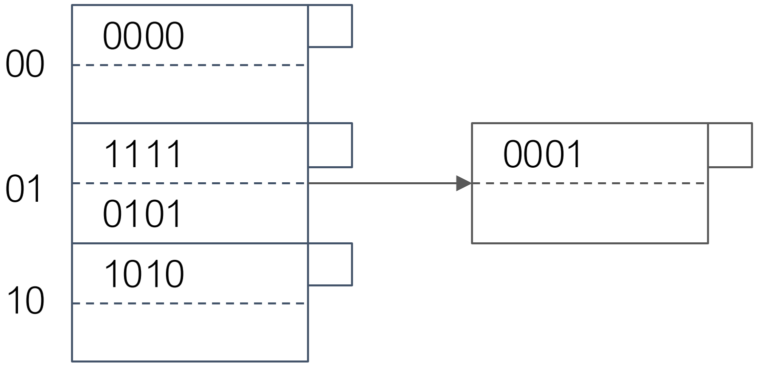


# Linear hash tables

- Continuing with example, add 0111.
  What happens here?

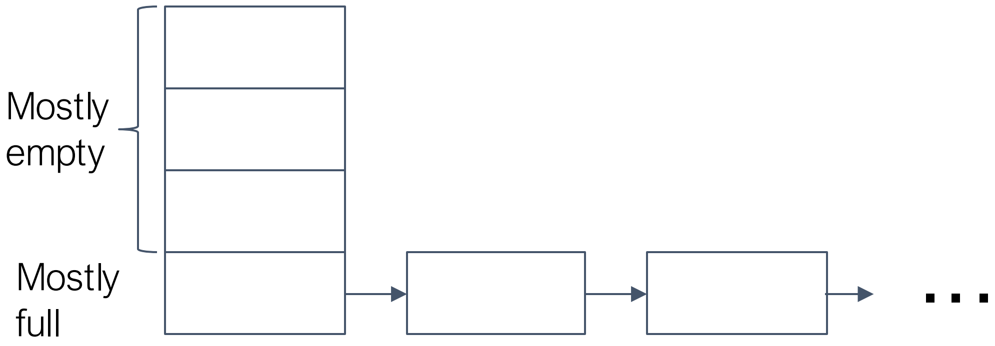
| | |
|---|---|
| # bits used | i = 2 |
| # buckets | n = 3 |
| # records | r = 5 |

Policy: limit r ≤ 1.7n



# Linear hashing summary

- Can grow table with little wasted space and avoiding full reorganizations
- Compared to extensible hashing, there is no array of buckets
- However, there can be a long chain of overflow blocks



# Multidimensional Indexes (14.4)

All the index structures discussed so far are one dimensional

- Assume a single search key, and they retrieve records that match a given search key value.
- The key can contain multiple attributes

Examples:

- KD-tree, R-tree